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CLASSICAL BLACK HOLE PRODUCTION IN QUANTUM PARTICLE COLLISIONS

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A semiclassical picture of black hole production in trans-Planckian collisions is reviewed.

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1. Introduction

(This talk was based on Refs. 1, 2, 3, where complete bibliography can be found.)

Let us compare two black hole (BH) formation processes, happening on vastly different physical scales. In one process, two solar mass BHs moving with relativistic velocities collide to form a bigger BH. In the other, a BH is formed in a collision of two trans-Planckian elementary particles. "Trans-Planckian" here means $E \gg 10^{19}$ GeV in standard 4-dimensional gravity, or $E \gg 1$ TeV in Large Extra Dimension scenarios of TeV-scale gravity.

The first, astrophysical, process is described by classical General Relativity. What about the second one? In particular, what is the role played by the quantum nature of elementary particles? To make these questions more precise, let us first review the classical gravity picture of BH production.

2. Classical gravity picture

In a totally classical description we consider a grazing collision of two fast point particles of energy $E \gg 1$ in D-dimensional Planck units (Fig. 1). Naively, we expect a BH to form if the impact parameter b is comparable to the Schwarzschild radius of a D-dimensional BH of mass E, where $D = 4 + n$, n being the number of

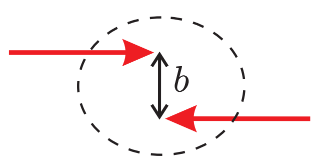

Fig. 1. Grazing collision.

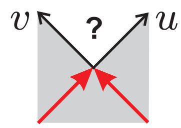

Fig. 2. Collision spacetime of two Aichelburg-Sexl waves (longitudinal slice).

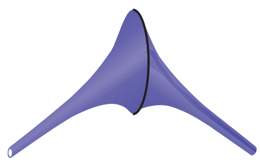

Fig. 3. Closed trapped surface.

large extra dimensions. In Planck units this corresponds to the condition:

$$b \lesssim R_h \sim E^{1/(D-3)} . \tag{1}$$

This naive picture can be made precise by studying the geometry of two colliding Aichelburg-Sexl shock waves, describing the fast point particles. The colliding shocks have curvature concentrated on the null planes $u = 0$ and $v = 0$ (Fig. 2). Spacetime is flat before and after the shocks. The interaction region $u, v > 0$ will be curved, and the metric there is unknown. To prove the BH formation, one can look for a closed trapped surface (CTS, also called apparent horizon) in the known part of spacetime. In Refs. 4, 5 such apparent horizons were indeed found for impact parameters in the range (1). They have a rather peculiar shape, consisting of two throats narrowing along the world lines of colliding particles and glued together at the transverse collision plane $u = v = 0$ (Fig. 3).

3. Why classical gravity is applicable

Two questions can be raised concerning the applicability of classical gravity in this problem:

- We are dealing with a potentially violent process occurring at Planckian energies. Shouldn't quantum gravity effects become significant?
- In accelerators, colliding particles are definitely not pointlike. Instead, they come in wide wavepackets of macroscopic size. Isn't this in direct contradiction with the point-particle approximation used in the above argument?

As it will turn out, in a more careful treatment these potential problems in a sense compensate each other, and the final result is not affected.

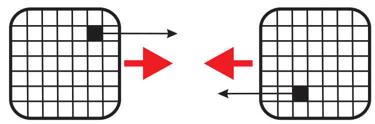

Fig. 4. Subdivided wavepackets

3.1. *Wavepacket subdivision*

The point-particle approximation used above cannot be taken over to quantum theory. Indeed, quantum description should be compatible with the usual position uncertainty, which for an ultrarelativistic particle is of the order of the wavelength E^{-1}. To incorporate this uncertainty we should consider wavepackets of finite spread $w \gtrsim E^{-1}$.

In fact, it is quite obvious that the CTS argument does not need particles to be exactly point-like, and works under a much milder assumption of them having size $w \ll R_h$. The resulting collision spacetime will be a small perturbation of the point-particle spacetime and will still contain a CTS ([3]).

However, the wavepackets describing particles in a collider beam have much larger, macroscopic size set by the beam radius ($\sim 10^{-3}$ cm at the LHC). To study BH production, these huge wavepackets should thus be subdivided into smaller wavepackets of size w satisfying the previous two limits:

$$E^{-1} \lesssim w \ll R_h \tag{2}$$

This subdivision (see Fig. 4) can be carried out in such a manner that different small wavepackets correspond to almost orthogonal states. (This orthogonality is obvious in position space, although it would be hard to see in the momentum space.) Because of this orthogonality, to compute the total BH production cross section, we must simply count the total number of pairs of small wavepackets, for which a BH forms (this is a 0-1 possibility). It is easy to see that such counting results in the geometric cross section.

By this argument we reduced the problem to analyzing collision of two wavepackets of size w. We will now show that there exist w satisfying (2), for which quantum gravity corrections to the semiclassical description are small.

3.2. *Curvature test*

For the classical solution to be stable with respect to quantum corrections, curvature must be small ($\ll 1$ in Planck units). The Riemann tensor of the Aichelburg-Sexl shock wave corresponding to the left-moving particle has δ-function components (x_i are $D-2$ transverse coordinates, $r = |x|$):

$$R_{uiuj} \propto E\,\delta(u)\,\partial_i\partial_j(1/r^{D-4})\ . \tag{3}$$

This field should be superposed with the similar field of the right-moving particle, shifted by b in the transverse direction. This right-moving shock wave will have

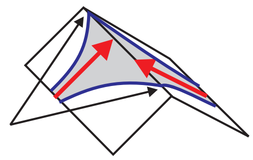

Fig. 5. A section of CTS from Fig. 3. The gray area will eventually become the BH interior.

large R_{vivj} components, and we can form a nonvanishing curvature invariant:

$$(R_{\mu\nu\lambda\sigma})^2 \sim E^{-\frac{2}{D-3}}\,\delta(u)\,\delta(v) \qquad (r \sim R_h). \tag{4}$$

This estimate would seem to imply that curvature blows up on the CTS at the points where it crosses the transverse collision plane (see Fig. 5). However, this is where the wavepacket width comes to the rescue. To take it into account, we have to smear out δ-functions in (4) on scale w. To keep integral equal 1, the effective maximal value of $\delta(u)$, $\delta(v)$ becomes $\sim 1/w$. Thus the final curvature estimate in the vicinity of the CTS takes the form [1,2,3]:

$$(R_{\mu\nu\lambda\sigma})^2 \sim R_h^{-2} w^{-2} \qquad (r \sim R_h). \tag{5}$$

From this we see that curvature can be kept small as long as $w \gg R_h^{-1}$ ([3]), a condition compatible with the allowed range (2).

3.3. *Graviton counting test*

Another necessary condition for the classical gravity description to be valid is that quantum fluctuations of the gravitational field have to be be suppressed compared to its classical value. This will happen when graviton occupation numbers of the colliding fields are large. The concept of gravitons should be applicable for linearized gravity, when deviation from the Minkowski metric is small:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1. \tag{6}$$

In this case $h_{\mu\nu}$ can be quantized as a free field, with quanta being transverse gravitons. In our case this condition turns out to be satisfied near the shock fronts.

Thus, we would like to count gravitons contained in the gravitational field of colliding particles, say, of the right-moving one. The presence of the other particle does not play a role in this counting before the shock waves collide. More specifically, we would like to count gravitons contained in the shock front at $r \sim R_h$, since this is the region relevant for the apparent horizon formation (see Fig. 6). A detailed calculation ([2]) shows that the spectrum of gravitons is given by

$$n_\omega \sim R_h^{D-4} \omega^{-3}. \tag{7}$$

This formula is valid up to frequencies $\omega \lesssim w^{-1}$, at which point the spectrum cuts off (see Fig. 7). The total number of gravitons with energy $\sim \Omega$ is

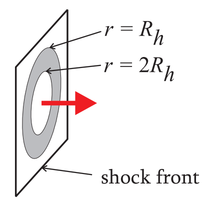

Fig. 6. Gravitons in the gray area need to be counted.

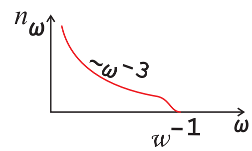

Fig. 7. Gravitons spectrum in the shock front.

$$N_\Omega \sim \int_{\omega\sim\Omega} d\omega\, n_\omega \sim R_h^{D-4}\Omega^{-2}. \tag{8}$$

Condition $N_\Omega \gg 1$ becomes most restrictive when applied at $\Omega \sim w^{-1}$, which is the smallest scale present in the classical solution. This implies that w has to satisfy $w \gg R_h^{2-D/2}$ ([3]). Again we see that this is compatible with (2).

4. Conclusions

In this talk we argued that it *is* possible to carry out the analysis of BH production in trans-Planckian elementary particle collisions in controlled semiclassical approximation. We showed that both criteria of semiclassicality (low curvatures, high quantum numbers) are satisfied. The geometric cross section estimate is in good shape.

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References

1. V. S. Rychkov, "Black hole production in particle collisions and higher curvature gravity," Phys. Rev. D **70**, 044003 (2004) [arXiv:hep-ph/0401116]
2. V. S. Rychkov, "Tests of classical gravity description for microscopic black hole production," arXiv:hep-ph/0405104
3. S. B. Giddings and V. S. Rychkov, "Black holes from colliding wavepackes," arXiv:hep-th/0409131
4. D. M. Eardley and S. B. Giddings, "Classical black hole production in high-energy collisions," Phys. Rev. D **66**, 044011 (2002) [arXiv:gr-qc/0201034].
5. H. Yoshino and Y. Nambu, "Black hole formation in the grazing collision of high-energy particles," Phys. Rev. D **67**, 024009 (2003) [arXiv:gr-qc/0209003].